REDEFINING RANDOMNESS
QUANTIS
WHEN RANDOM NUMBERS CANNOT BE LEFT TO CHANCE

## RANDOMNESS TEST REPORT

## Version 2.0

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## REDEFINING RANDOMNESS

## QUANTIS RANDOMNESS TEST REPORT

Quantis-OEM-4M is a physical random number generator (RNG) component that produces truly random numbers at a high bit rate of $4 \mathrm{Mbits} / \mathrm{sec}$. The randomness of the number generation process is guaranteed by the laws of quantum physics. The component is available as such or mounted as a USB device, on a PCI Express board and or on a PCI board.

## INTRODUCTION TO RANDOM NUMBER GENERATORS

Random number generators (RNGs) can be grouped in three families:

## Pseudo-random number generators

Software-based generators use an algorithm into which some initial value is fed to produce a sequence of pseudorandom numbers. Since the sequence they produce is always periodic, they should not be used in applications where randomness is required.

Physical random number generators based on a chaotic process
Macroscopic processes described by classical physics can be used to generate random numbers. Typical chaotic processes include the monitoring of some electric noise current in a resistor or in a diode. This current is not random, but just very complex to describe. Determinism is hidden behind complexity. Although their random numbers are likely to pass randomness tests, these generators are difficult to model. This implies that it is impossible to verify if they are operating properly while acquiring numbers. In addition, it is difficult to ensure that the system is not interacting with environment parameters like temperature or an electromagnetic field.

## - Physical random number generators based on a quantum process

Quantum physics is the only theory within the fabric of modern physics that integrates randomness. This fact was very disturbing to physicists like Einstein who invented quantum physics. However, its intrinsic randomness has been confirmed over and over again by theoretical and experimental research. Truly random numbers can be generated by monitoring radioactive decay of heavy elements. Although they produce numbers of excellent quality, such generators are not suitable for commercial applications. In 2001, id Quantique introduced the first commercial physical random number generator based on quantum physics. The randomness is guaranteed by the random behaviour of single 'light particles' - called photons - hitting a semi-transparent mirror. The process by which photons incident on such a component is either reflected or transmitted is intrinsically random and cannot be influenced by external parameters. The quantum process used to generate the random bits, its immunity to external factors, plus the fact that the Quantis unit performs live verifications of its internal functions, guarantees a high level of trust in the random numbers that are generated.

This document gives a brief overview of the tests that were performed to evaluate the randomness of the bit stream generated by the Quantis product. The most stringent randomness tests, namely the NIST test (issued by the National Institute of Standards and Technology, Special Publication 800-22) and the DIEHARD test, were performed. The Quantis random number generator successfully passed all randomness tests.

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## RANDOMNESS TEST SUITE 1: NIST (http://csrc.nist.gov/rng/)

The NIST statistical test suite for random number generators offers a battery of 16 statistical tests. These tests assess the presence of a pattern which, if detected, would indicate that the sequence is non-random. The properties of a random sequence can be described in terms of probability. In each test a probability, called the pvalue, is extracted. This value summarizes the strength of the evidence against the perfect randomness hypothesis. A p-value of zero indicates that the sequence appears to be completely non-random. A p-value larger than 0.01 means that the sequence is considered to be random with a confidence of $99 \%$.

The following table shows typical results obtained on a series of 1 billion bits. The random data was generated by a Quantis module in about 4 minutes. The NIST test suite was performed on 1000 bit streams, each stream containing 1 million random bits. The mean and variance of the $p$-values are shown below:

| Testname | Mean of <br> p-value | Variance | Conclusion |
| :--- | :---: | :---: | :---: |
| Approximate Entropy Test | 0.489 | 0.088 | SUCCESS |
| Frequency Test within a Block | 0.506 | 0.081 | SUCCESS |
| Cumulative Sums Test | 0.499 | 0.081 | SUCCESS |
| Discrete Fourier Transform (Spectral) Test | 0.493 | 0.079 | SUCCESS |
| Binary Matrix Rank Test | 0.498 | 0.084 | SUCCESS |
| Run Test | 0.497 | 0.081 | SUCCESS |
| Serial Test | 0.495 | 0.078 | SUCCESS |
| Maurer's Universal Statistical Test | 0.493 | 0.081 | SUCCESS |
| Linear Complexity Test | 0.499 | 0.083 | SUCCESS |
| Test for the Longest Run of Ones in a Block | 0.503 | 0.087 | SUCCESS |
| Non-overlapping Template Matching Test | 0.499 | 0.082 | SUCCESS |
| Overlapping Template Matching Test | 0.490 | 0.081 | SUCCESS |
| Frequency (Monobit) Test | 0.505 | 0.084 | SUCCESS |
| Lempel-Ziv Compression Test | 0.480 | 0.080 | SUCCESS |
| Random Excursions Test | 0.503 | 0.083 | SUCCESS |
| Random Excursions Variant Test | 0.502 | 0.082 | SUCCESS |



The significance level for each test of the NIST SP800-22 test suite is set to $1 \%$. A value called the proportion is extracted for each test. The proportion is the number of sequences having a $p$-value larger than $1 \%$, divided by the
number of binary sequences. For this test 1000 binary sequences of 1 million bits each was used.

The plot on the right shows the proportion for each of the 16
tests. NIST SP800-22 specifies a range of acceptable proportions. Since the proportion for each test is larger than
0.9805 and smaller than 0.9994 , there is very high confidence that the data is truly random.

Further analysis has also confirmed that the distribution of the $p$-values is within the NIST SP800-22 specifications.

The bit stream generated by Quantis has therefore a high level of randomness.

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## RANDOMNESS TEST SUITE 2: DIEHARD (http://www.csis.hku.hk/~diehard/cdrom/)

The DIEHARD test suite is a battery of 18 stringent statistical tests. Large files of 1 billion bits each were generated with a Quantis module and the randomness of the data was tested using the DIEHARD test suite. The following table shows a typical result obtained on a single bit stream of 1 billion bits. A p-value larger than 0.01 and smaller than 0.99 means that the sequence is random with a confidence of $99 \%$.
All the tests that were performed on the Quantis modules have confirmed the high level of confidence in the randomness of the data.

| Test name | p-value | Conclusion |
| :--- | :---: | :---: |
| Birthday Spacing Test | 0.493 | SUCCESS |
| Overlapping 5-Permutation Test | 0.679 | SUCCESS |
| Binary Rank Test (31×31 matrices) | 0.692 | SUCCESS |
| Binary Rank Test (32x32 matrices) | 0.789 | SUCCESS |
| Binary Rank Test (6x8 matrices) | 0.730 | SUCCESS |
| Bitstream Test | 0.622 | SUCCESS |
| Overlapping-Pairs-Sparse Occupancy Test | 0.524 | SUCCESS |
| Overlapping-Quadruples-Sparse-Occupancy Test | 0.562 | SUCCESS |
| DNA Test | 0.657 | SUCCESS |
| Count-the-1's Test (on stream of bytes) | 0.418 | SUCCESS |
| Count-the-1's Test (on specific bytes) | 0.557 | SUCCESS |
| Parking Lot Test | 0.409 | SUCCESS |
| Minimum Distance Test | 0.551 | SUCCESS |
| 3D Spheres Test | 0.681 | SUCCESS |
| Squeze Test | 0.415 | SUCCESS |
| Overlapping Sums Test | 0.460 | SUCCESS |
| Runs Test | 0.698 | SUCCESS |
| Craps Test | 0.696 | SUCCESS |



## RANDOMNESS TESTS IN PRODUCTION

As part of our quality insurance program, all QuantisOEM modules and Quantis-PCI cards undergo a series of randomness tests before shipment to customers. The following randomness tests are being performed:

- Balancing Test
- Autocorrelation Test
- Maurer Universal Test
- Run Test
- Single level Serial Test

■ Single Level Entropy Test

- Single Level Frequency Test

ORDERING INFORMATION

■ Quantis-OEM-4M

- Quantis-PCle-4M
- Quantis-USB-4M
- Quantis-PCI-1
- Quantis-PCI-4

OEM component generating a random bit stream of $4 \mathrm{Mbits} / \mathrm{s}$ PCI Express card with 1 module generating a random bit stream of $4 \mathrm{Mbits} / \mathrm{s}$ USB device with 1 module generating a random bit stream of $4 \mathrm{Mbits} / \mathrm{s}$ PCI card with 1 module generating a random bit stream of $4 \mathrm{Mbits} / \mathrm{s}$ PCI card with 4 modules generating a random bit stream of $16 \mathrm{Mbits} / \mathrm{s}$

